MTH 201: Multivariable Calculus and Differential Equations

Problem Set 2: Line Integrals

1. Integrate the scalar field f over the given curve C.
   (a) $f(x,y,z) = xy + y + z$, $C : r(t) = 2ti + tj + (2-2t)k$, $k \in [0,1]$.
   (b) $f(x,y,z) = \frac{\sqrt{3}}{x^2+y^2+z^2}$, $C : r(t) = ti + tj + tk$, $t \in [1,\infty)$.
   (c) $f(x,y,z) = -\sqrt{x^2+z^2}$, $r(t) = (a\cos t)j + (a\sin t)k$, $t \in [0,2\pi]$.
   (d) $f(x,y) = \frac{x^2}{y^{4/3}}$, $C : x = t^2, y = t^3$, $t \in [1,2]$.
   (e) $f(x,y) = x^2 - y$, $C : x^2 + y^2 = 4$ in the first quadrant from $(2,0)$ to $(0,2)$.
   (f) $f(x,y) = \frac{x+y^2}{\sqrt{1+x^2}}$, $C : y = x^2/2$ from $(1,.5)$ to $(0,0)$.
2. Integrate the vector field F over the given curve C.
   (a) $F(x,y,z) = xyi + yzj + xzk$, $C : r(t) = ti + tj + tk$, $t \in [0,1]$.
   (b) $F(x,y,z) = (y+z)i + (z+x)j + (x+y)k$, $C : r(t) = ti + t^2j + t^4k$, $t \in [0,1]$.
   (c) $F(x,y,z) = \frac{1}{x^2+2}j$, $C : r(t) = ti + t^2j + t^4k$, $t \in [0,1]$.
3. Find the work done by F over the curve C.
   (a) $F(x,y,z) = 2yi + 3xj + (x+y)k$, $r(t) = (\cos t)i + (\sin t)j + (t/6)k$, $t \in [0,2\pi]$.
   (b) $F(x,y,z) = zi + xj + yk$, $r(t) = (\sin t)i + (\cos t)j + tk$, $k \in [0,2\pi]$.
4. Evaluate the following line integrals in the plane.
   (a) $\int_C (x-y)\,dx + (x+y)\,dy$, C : counterclockwise around the triangle with vertices $(0,0)$, $(1,0)$, and $(0,1)$.
   (b) $\int_C F \cdot T\,ds$, where $F(x,y) = x^2i - yj$ and $C : x = y^2$ from $(4,2)$ to $(1,-1)$.
5. Find the flow of the velocity vector field F along the given curve C.
   (a) $F(x,y,z) = x^2i + yzj + y^2k$, $C : r(t) = 3tj + 4tk$, $t \in [0,1]$.
   (b) $F(x,y,z) = -yi + xj + 2k$, $C : r(t) = (-2\cos t)i + (s\sin t)j + 2tk$, $t \in [0,\pi]$.
   (c) $F(x,y) = (x+y)i - (x^2+y^2)j$, C : upper half of $x^2 + y^2 = 1$ from $(1,0)$ to $(-1,0)$.
   (d) $F(x,y) = (x+y)i - (x^2+y^2)j$, C : the line segment from $(1,0)$ to $(0,-1)$ followed by the line segment from $(0,-1)$ to $(-1,0)$.
6. Find the flux of following vector fields across C.
   (a) $F(x,y) = -yi + xj$, $C : r(t) = (\cos t)i + (4\sin t)j$, $t \in [0,2\pi]$.
   (b) $F(x,y) = 2xi + (x-y)j$, $C : r(t) = (a\cos t)i + (a\sin t)j$, $t \in [0,\pi]$.
   (c) $F(x,y) = (x+y)i + (x^2+y^2)j$, C : outward across triangle with vertices $(1,0)$, $(0,1)$, and $(-1,0)$.